1. Prove by induction:

\[ 2 + 5 + 8 + \cdots + (3n - 1) = \frac{n(3n + 1)}{2}, \text{ for } n > 0. \]

2. If \( u \) and \( v \) are strings, \( u, v \in \Sigma^* \), prove that \( (uv)^R = v^Ru^R \), by induction on the length of \( v \).

3. Define:
   (i) an equivalence relation;
   (ii) a bijection;
   (iii) a countable set;
   (iv) a language over an alphabet \( \Sigma \);
   (v) (give a recursive definition of:) the length of a string;
   (vi) (give a recursive definition of:) a regular set, and of a regular expression (\textit{Basis:} What are the primitive regular expressions? \textit{Recursive step?} \textit{Closure:} Explain.
   (vii) a DFA (give formal definition: as a 5-tuple; explain all elements of the 5-tuple, including the mapping of the transition function \( \delta \));
   (viii) a regular grammar, a CFL.

4. Consider the set \( A \) of all total functions which map from the set \( \mathcal{N} \) of the natural numbers to the set \( \{0, 1\} \) of the two elements 0 and 1:

\[ A = \{ f \text{ total function} \mid f : \mathcal{N} \to \{0, 1\} \}. \]

Prove by (Cantor) diagonalization that the set \( A \) is uncountable.

5. Prove that the set \( \mathcal{Z} \) of all integers,

\[ \mathcal{Z} = \{ \cdots, -3, -2, -1, 0, 1, 2, 3, \cdots \} \]

is countably infinite.
6. (i) Let $X$ be a set. Define:
   a) $X^*$
   b) $X^+$

(ii) Let $X = \{a, b, c\}$; $Y = \{ccb, d\}$.
   Express:
   a) $XY$
   b) $X^0$
   c) $XX = X^2$

7. Exercise: Show that the set of the regular languages is closed under union, concatenation and Kleene closure.

8. For a given DFA and a particular string, give a computation on the string. Is the string accepted or rejected?
   Give an algorithm to simulate a DFA. Work through it for an example DFA and string. What is the time complexity of the algorithm as a function of the length of the string?

9. Give an overview of the Chomsky hierarchy of languages and their grammars, with the characteristic form of the grammar rules for each type.